

Lesson Title: Arranging Yummy Doggy Treats in Boxes

Foundation 4: Problem Solving and the Mathematical Practices

Mathematics Grade 4

Mathematics Common Core State Standards

Domain: Operations and Algebraic Thinking 4.OA

- *Cluster: Gain familiarity with factors and multiples*
4.OA.4—Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.
- *Cluster: Generate and analyze patterns*
4.OA. 5—Generate a number or a shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself.

Common Core State Mathematical Practices

MP1—Make sense of problems and persevere in solving them.

MP2—Reason abstractly and quantitatively.

MP3—Construct viable arguments and critique the reasoning of others.

MP4—Model with mathematics.

MP5—Use appropriate tools strategically.

MP6—Attend to precision.

MP7—Look for and make use of structure.

Materials

- square tiles
- red, green, black marker for each student
- graph paper (1 cm)

- Arranging Yummy Doggy Treats in Boxes Problem
- Arranging Yummy Doggy Treats in Boxes, Table of Treats Recording Sheets (Student sheets 1,2,3,4,5)

Problem Solving Strategies

- Make a model
- Make a table
- Find a pattern

Vocabulary

As these words are discussed, they should be added to your mathematics word wall.

- factor
- multiple
- product
- even*
- odd*
- array
- rectangle*
- square*
- prime
- composite
- dimension

* review vocabulary

Lesson Overview

1. This lesson incorporates the content standards with the mathematical practices, and at the same time develops specific problem solving strategies to build a conceptual understanding of the key ideas of multiplication including factors and products. The lesson also connects those ideas to identifying prime and composite numbers.
2. This lesson, which will take several days to complete, is divided into three parts. The first part will **introduce** students to the vocabulary and review the mathematical concepts and skills necessary to approach the new concepts. The inquiry-based problem is then presented to students with enough information to enable them to have entry into approaching the task. The second part of the lesson will give students time to **explore** and work on the task in small groups. The third part of the lesson brings students back together to **summarize and discuss** their findings and solutions. This third part of the lesson helps students to make connections among the mathematical ideas of the lesson and to share the strategies.
3. The lesson and its extensions focus on the key concepts of finding factors and multiples of numbers as well as recognizing prime and composite numbers. Although the lesson focuses on the numbers up to 40, it can be extended to 100 without the use of the physical materials. Students make connections from their work with various representations to identifying the special nature of numbers identified as **prime** and numbers identified as **composite**. Additionally through this task, students understand why the number 1 is not considered to be prime or composite.
4. **Differentiation:** Some students may depend on the square tiles to find the dimensions of rectangles and connecting the dimensions to the factors of the number of dog treats. Others may immediately recognize that the dimensions are the same as the factors and proceed with less dependence on the physical models. Others may want to record their work on graph paper rather than to work with the tiles. Part of the role of the teacher is to help students identify which representations are most helpful. Most students will recognize that the rectangles are the same as the multiplication facts. However some possible solutions may not be basic facts (for example a 2 x 14 can be made from 28

treats.) In these cases, students may need to use the concrete materials in order to find all of the possible boxes.

Addressing the Common Core Mathematical Practices

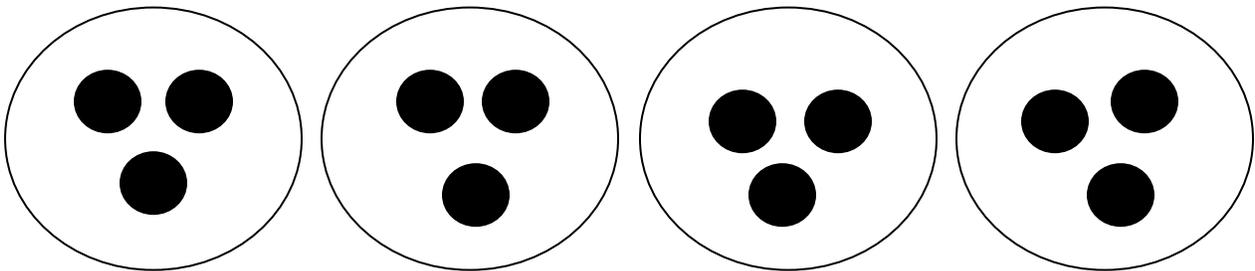
- **MP1**—Make sense of problems and persevere in solving them.
Students discover prime and composite numbers as they complete a task to find the factors of a given number.
- **MP2**—Reason abstractly and quantitatively.
To complete the questions, students must reason and make sense of the task.
- **MP3**—Construct viable arguments and critique the reasoning of others.
Students use patterns to draw conclusions and explain their thinking to others. Teacher questions and other student explanations help students to gain a clear understanding of the mathematical concepts.
- **MP4**—Model with mathematics.
Students use concrete materials, pictures and tables to do mathematics.
- **MP5**—Use appropriate tools strategically.
Students may choose for which numbers they want to use the tiles. They can also draw the rectangles on graph paper if they prefer.
- **MP6**—Attend to precision.
Students use appropriate and precise vocabulary as they discuss the concepts throughout the task. They also must be sure they have found all of the possible factor pairs for each number.
- **MP7**—Look for and make use of structure.
By looking for patterns, students discover underlying structure of prime and composite numbers and their relationships to factors (part one of lesson) and multiples (part two of the lesson).

Part I: Introduction

1. Write $4 \times 3 = 12$ on the board.
2. Ask students to show what this equation represents using pictures and words.

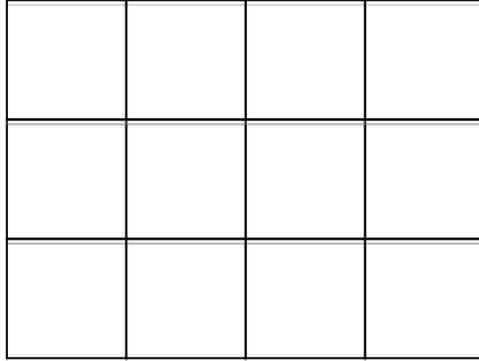
Sample Representations of 4×3

Set model:



Words: 4 groups of 3 dots are 12 dots in all

Area model



Words: 4 rows of tiles with 3 in each row equals 12 tiles.

3. Introduce vocabulary to students. Explain that in the equation $4 \times 3 = 12$, the numbers that are multiplied are called *factors* and the answer, or total number, is called the *product*

4. Have students discuss the following terms and questions in pairs and record the answers in their problem-solving journals.

- So in $4 \times 3 = 12$, 4 and 3 are the *factors* and 12 is the *product*.
- 12 is also called a *multiple* of 3 because I can multiply a number by 3 and get 12.
- Is 12 a multiple of 4? (*Yes because I can multiply a number by 4 and get 12.*)
- What are some other multiples of 3?
(*Answers will vary, but may include 6, 9, 15, 18, 21, 30, etc.*)
- Give the first 8 multiples of 2. (*0, 2, 4, 6, 8, 10*)

4. Write the following sentence frames on the board and ask students to complete the frames with their partner. Then have them record the completed sentence frames in problem-solving journals.

- *A multiple is the same as a _____, for example _____*
- *A product is the same as a _____, for example _____*

5. Ask students, what are some other multiplication equations with a product of 12? Have students discuss in pairs and record the equations in their problem-solving journal. Then have students share out the equations with the whole group. Record the equations that students identify.

6. Ask: How can we be sure we have found them all? (Organize the equations from least to greatest first factor).

$1 \times 12 = 12$

$4 \times 3 = 12$

$2 \times 6 = 12$

$6 \times 2 = 12$

$3 \times 4 = 12$

$12 \times 1 = 12$

7. Ask the students to describe any patterns they notice in the list of factor pairs. Ask them to name all of the factors of 12 (1, 2, 3, 4, 6, 12).

8. Repeat steps 2 through 5 for to find the factors for 10. Find the equations for 10 using pictures, numbers and words and then list the factors of 10.

$1 \times 10 = 10$

(1 group of 10)

$2 \times 5 = 10$

(2 groups of 5)

$5 \times 2 = 10$

(5 groups of 2)

$10 \times 1 = 10$

(10 groups of 1)

The factors of 10 are 1, 2, 5, and 10

Part II: Explore

The Task:

1. Place students in groups of 3 or 4 to work on the problem. Give each student the *Arranging Yummy Doggy Treats* problem and the *Table of Treats*. Do not pass out the blank tables, however, as they are for students who finish 1 to 40 and have time to do additional boxes.

2. Ask students to read the problem and use tiles to show all the possible ways that 6 doggy treats can be arranged in the base of a box. Talk about the dimensions of the base of each box. Find the dimensions of the base of the last 2 boxes (2 x 3 and 3 x 2). Demonstrate how to complete the table.

3. Have students work in groups to find the way that 12 and 10 doggy treats can be arranged in the box base from the work they did in the introduction part of the lesson and complete the table for those boxes as well.

- Ask students how the factors of 12 and 10 that they listed in their problem-solving journals relate to the dimensions of the doggy treat arrangements.

4. After they find the factors for each number, they should also list the first 5 non-zero multiples for the numbers from 1 to 20.
5. Before the students begin to work in their groups ask them to use tiles to find all the possible dimensions of the bases of boxes for 4 treats.
 - At this point many students may successfully find 1×4 and 4×1 but may not find that 2×2 is an acceptable response. Continue to probe, until someone builds a box base that is 2×2 . Some students will say that is a square and not a rectangle. If no one builds a 2×2 base, build one and ask if it is a correct response. Some students will reply that it is not correct because it is a square. Ask students to define a square and to define a rectangle (a *square* is a figure with 4 right angles and 4 congruent sides; a *rectangle* is a figure with 4 right angles and 4 sides).
 - It is important that students realize that a square is a special kind of rectangle because it has 4 right angles and 4 sides. It is a special rectangle because the sides are also congruent.
6. Students should work to complete the table for 1 to 40 treats.

Differentiation strategies:

1. (*Below-Level Students Tip 1*) Watch for students who are reluctant to use the tiles but need them in order to find all of the possible combinations. Some students will only find the boxes that are related to the multiplication facts. By using the tiles and trying other combinations, they can find all of the possible solutions.
2. (*Below-Level Students, Tip 2*) Students who struggle to identify the multiples of numbers greater than 12 might be given a calculator. Encourage them to use various strategies (such as adding on or the hundreds chart) to find the multiples.
3. (*Below-Level Students, Tip 3*) some students may find drawing the bases of the boxes on graph paper to be helpful. Using 1 cm graph paper and colored markers will help them to keep track of their work.
4. (*On-Level Students/Above-Level Students, Tip 1*) Those students who finish early can continue to 60 treats if they have found the bases of boxes for 1 to 40.
5. (*On-Level/Above-Level Students, Tip 2*) Have students find multiples for numbers greater than 20 and discover patterns in listing multiples.

Part III: Summarize and Discuss

Check student work as they complete the table for up to 40 tiles. Those who finish may continue with a greater number of treats. When all have completed the table for up to 40, give students the discussion questions. They can work in groups to answer the questions or you may choose to have them discuss the questions together as a class.

It is important to give students time to think about and answer each question before you discuss it so that they have time to draw some conclusions about the numbers and patterns rather than just copy the work of others. This is a good time to use the think-share-pair strategy.

The teacher solution sheet has sample questions that will probe student reasoning and thinking beyond just getting the right answer. Of course you will determine other questions as students have additional responses. What is important here is for you to let student's correct misconceptions by thinking more deeply about their answers rather than for you to tell them the correct answer.

A great activity following this lesson is to start a class or find some time each day to ask students who can recite all of the prime numbers to 40 (or beyond) or who can recite all of the composite numbers to 40 (or beyond). Students enjoy the challenge of this ongoing review.

Assessment

Formative Assessment: Using formative assessment strategies throughout the task (such as questioning, observing students as they complete the task, and reviewing written work) will enable you to determine if students understand the concepts and allow you to take immediate steps to clear up any misconceptions as well as to see if students have the prerequisite skills to complete the task.

Prerequisites: Do students understand multiplication as the process of taking x groups and putting y objects in each group to find the total number of objects? Can students model various multiplication equations with concrete and pictorial representations? Do student know multiplication facts?

This task will reinforce those concepts as well as extend student knowledge and understanding to recognize new sets of numbers (primes and composites) by recognizing patterns among their factors.

Observation Checklist of Specific Concepts/Skills to be developed: The teacher's role throughout the lesson is to carefully observe and take notes on whether students are demonstrating a working knowledge of the following areas:

1. As new vocabulary is developed, provide students with opportunities throughout discussions and in their writing to make use of that vocabulary. Specific focus should be on the terms *factor*, (the factors of 10 are 1, 2, 5, and 10), *multiple*, *prime* and *composite*.
2. Observe to see if students can model all of the factors for the given number. Are they familiar with a model (e.g, a set model or array model) to represent a given multiplication fact. Can they connect the model to words?
3. As students work in groups to complete the task, are they using specific problem solving strategies as they approach the task? How are they certain they have all of the possible boxes or factors? If they do not find all of the combinations, particularly for larger numbers, do they use tools appropriately to find less obvious possibilities?
4. Is student work organized in a way that will help them to analyze and draw conclusions?
5. During the discussion, can students explain their reasoning and respond to other strategies or solutions by agreeing or asking questions?
6. Is student written work complete?
7. Provide an exit task such as the following:

In the multiplication sentence $5 \times 7 = 35$,

5 is a _____

35 is a _____

7 is a _____

Arranging Yummy Doggy Treats in Boxes

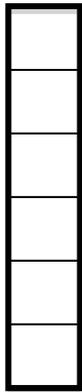
Summative Assessment

Name: _____

Date: _____

Yummy Doggy Treats are square in shape. They are sold in rectangular boxes with one layer of treats in a box.

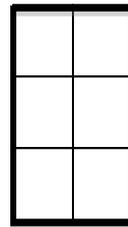
A box of six treats could be arranged in any of the following shapes.



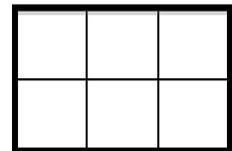
1 by 6



6 by 1



___ by ___



___ by ___

The numbers under each box show the *dimensions* of the base of the box. The first box on the left is 1 treat wide and 6 treats high or 1 by 6. The second box is 6 treats wide and 1 treat high.

1. Write the dimensions of the other two boxes.

2. Our class has been hired to find all of the possible ways Yummy Doggy Treats from 1 to 40 could be arranged in boxes. Work in your groups to complete the table of treats. Look for patterns as you complete the table.