

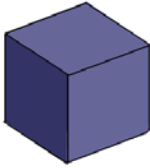
The Painted Cube Problem

Name: _____

Date: _____

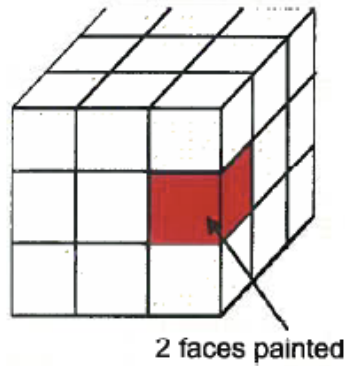
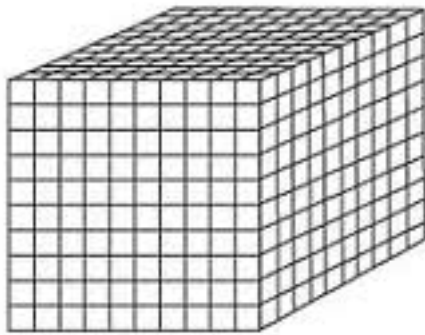
Directions: Read and solve the problem below.

Mark works for the Beautiful Block Company. His job is to paint the cubes that lucky boys and girls receive for their birthdays to and use as blocks to build beautiful structures!



Mark's job is to take the small wooden cubes and dip them completely into a can of paint. Notice how every face of the cube is covered in paint.

One day Mark has a great idea. He decides to glue the small cubes together to make a $10 \times 10 \times 10$ cube and then dunk the large cube into the paint. He thinks using this method he can get his work done faster. He makes several large cubes, paints them and leaves them to dry.



Mark is so proud of himself. No one has ever painted this many cubes in one day! He is certain his boss will give him a raise when she sees all of the work Mark has done!

However, the next day Mark arrives at work and finds the glue didn't stick and the large cubes have all fallen apart. What's even worse is that the cubes are not completely painted. "What was I thinking?" Mark sobbed.

Mark now has a lot of small cubes that do not have all 6 faces painted. How many cubes have 1 face painted, 2 faces painted, 3 faces painted, or 0 faces painted?

The Painted Cube Problem Student Recording Sheet

Directions: The table below has simpler problems that can be used to solve the more complex Painted Cube Problem. Use the table to help you organize your work from solving simpler problems. Then use the patterns from the simpler problem to solve the Painted Cube Problem. Follow the steps below to complete the table.

1. Determine the number of small cubes that made up each of the larger cube. Record the number of smaller cubes in the first column of the table.
2. For each large cube, determine the number of smaller cubes that have paint on 6 faces, 5 faces, 4 faces, 3 faces, 2 faces, 1 face, 0 faces. Record the answers in the table.

Large Cube Dimensions	Total Number of Small Cubes	Number of Cubes with...						
		6 Faces Painted	5 Faces Painted	4 Faces Painted	3 Faces Painted	2 Faces Painted	1 Face Painted	0 Faces Painted
2 x 2 x 2								
3 x 3 x 3								
4 x 4 x 4								
5 x 5 x 5								
6 x 6 x 6								
7 x 7 x 7								
8 x 8 x 8								
9 x 9 x 9								
10 x 10 x 10								
<i>n x n x n</i>								

Earning an Allowance Student Assessment

Directions: Solve the problem and answer the questions below.

Alex, who always has a scheme for everything, has just made a deal with his parents. He told them that rather than paying a weekly allowance, they can pay him for doing his chores in the following way. He will receive one cent on the first day, two cents on the second day, four cents on the third day, and so on. Each day's pay will be double the pay of the previous day. How much money will Alex's parent pay him on the 21st day? What is the total amount he will have been paid by that time?

1. Graph the amount Alex will earn each day and the total income accrued each day. What do you notice about the graph?
2. Describe the pattern for the amount of money earned each day and the total amount of money accrued each day. Write an algebraic expression to represent the total amount of money accrued each day as it relates to the money earned on that day.

3. How much money will Alex's parent pay him on the 21st day?

4. What is the total amount of money will Alex have earned by the 21st day?

Answer Key for Painted Cube Student Recording Sheet

Large Cube Dimensions	Total Number of Small Cubes	6 Faces Painted	5 Faces Painted	4 Faces Painted	3 Faces Painted	2 Faces Painted	1 Face Painted	0 Faces Painted
2 x 2 x 2	8	0	0	0	8	0	0	0
3 x 3 x 3	27	0	0	0	8	12	6	1
4 x 4 x 4	64	0	0	0	8	24	24	8
5 x 5 x 5	125	0	0	0	8	36	54	27
6 x 6 x 6	216	0	0	0	8	48	96	64
7 x 7 x 7	343	0	0	0	8	60	150	125
8 x 8 x 8	512	0	0	0	8	72	216	216
9 x 9 x 9	729	0	0	0	8	84	294	343
10 x 10 x 10	1000	0	0	0	8	96	384	512
$n \times n \times n$	n^3	0	0	0	8	$12(n-2)$	$6(n-2)^2$	$(n-2)^3$

Earning an Allowance Student Assessment Answer Key

Alex, who always has a scheme for everything, has just made a deal with his parents. He told them that rather than paying a weekly allowance, they can pay him for doing his chores in the following way. He will receive one cent on the first day, two cents on the second day, four cents on the third day, and so on with each day's pay becoming double the pay of the previous day. How much money will Alex's parent pay him on the 21st day? What is the total amount he will have been paid by that time?

1. Graph the amount Alex will earn each day and the total income earned. What do you notice about the graph?

Day	Amount Earned Each Day	Total Income Earned
1	\$0.01	\$0.01
2	\$0.02	\$0.03
3	\$0.04	\$0.07
4	\$0.08	\$0.15
5	\$0.16	\$0.31
6	\$0.32	\$0.63
7	\$0.64	\$1.27
8	\$1.28	\$2.55
9	\$2.56	\$5.11
10	\$5.12	\$10.23
11	\$10.24	\$20.47
12	\$20.48	\$40.95
13	\$40.96	\$81.91
14	\$81.92	\$163.83
15	\$163.84	\$327.67
16	\$327.68	\$655.35
17	\$655.36	\$1,310.71
18	\$1,310.72	\$2,621.43
19	\$2,621.44	\$5,242.87
20	\$5,242.88	\$10,485.75
21	\$10,485.76	\$20,971.51

2. Describe the pattern for the amount of money earned each day and the total amount of money accrued each day. Write an algebraic expression to represent the total amount of money accrued each day as it relates to the money earned on that day.

Each day's allowance is double the previous day. The total amount on any day is \$.01 less than the amount he will be paid on the next day. Some students may notice that calculating 2^{n-1} will give the amount of money earned on any day.

3. How much money will Alex's parent pay him on the 21st day?
On the 21st day, Alex will be paid \$10,485.76.
4. What is the total amount of money will Alex have earned by the 21st day?
Alex will have made a total of \$20,971.51 and his parents will be very unhappy!